

Plenary talks

Martin Bauer: *Smooth perturbations of the functional calculus and applications to Riemannian geometry on spaces of mappings.*

I will give a short introduction to selected topics of mathematical shape analysis. In particular I will discuss several metrics on manifolds of mappings that are of interest for applications in this field. An important question, that arises in this context concerns the existence and uniqueness of length minimizing deformations. These deformations can be (locally) described by the so-called geodesic equation. Proving local and global well-posedness of these equations using the framework of Ebin and Marsden requires one to study pseudo differential operators with non-smooth coefficients and in particular one needs to show smooth dependence on the coefficient functions. In the second part I will discuss a very general result in this direction, that is based on representing non-linear functions of an operators via functional calculus.

Theodore Drivas: *Singularities, dissipation and unpredictability in Onsager's Theory of "Ideal" Turbulence.*

We describe some recent advances in mathematical turbulence theory, with a focus on the observable phenomena of enhanced dissipation, strong mixing, and unpredictability.

Taryn Flock: *A nonlinear Brascamp-Lieb inequality.*

Brascamp-Lieb inequality generalizes many inequalities in analysis, including the Hölder, Loomis-Whitney, and Young's convolution inequalities. The focus of the talk will be a nonlinear generalization of the classical Brascamp-Lieb inequality in a general setting. As a corollary, we show that the best constant in Young's convolution inequality in a small neighborhood of the identity of a general Lie group, approaches the euclidean constant as the size of the neighborhood approaches zero.

A first step in this analysis is understanding the regularity of the sharp constant in the Brascamp-Lieb constant. This work has other applications including a multilinear Kakeya-type inequality which is used in Bourgain, Demeter's, Guth's proof of the Vinogradov mean value theorem from number theory. The Brascamp-Lieb constant also makes a surprise appearance in operator scaling, a generalization of the well-known matrix scaling algorithm from computer science. Time permitting, we will discuss these connections as well. (Joint work with Jon Bennett, Neal Bez, Stefan Buschenhenke, Michael Cowling, and Sanghyuk Lee.)

Sinan Gunturk: *Extracting bits from analog samples: frames and beyond.*

Sampling theorems provide the first step for obtaining digital representations of analog signals, the second step being the quantization of these samples. But it is often not obvious how to carry out the quantization step in order to achieve the best rate-distortion trade off possible, especially in the presence of redundancy. We will present a general approach called "distributed beta encoding" which can achieve superior (and often near-optimal) rate-distortion performance in a wide variety of sampling scenarios. These will include Fourier and Gabor sampling, and also some nonlinear ones such as compressive and phaseless sampling.

Alex Iosevich: *TBA.*

Mihalis Kolountzakis: *Bases of exponentials, tiling by translation and the zeros of the Fourier Transform.*

Over the past several decades mathematicians have been trying to understand which domains admit an orthogonal (or, sometimes, not) basis of exponentials of the form $e_{\lambda}(x) = e^{2\pi i \lambda x}$, for some set of frequencies Λ . (This sometimes goes by the name *non-harmonic* Fourier Analysis.) It is well known that

we can do so for the cube, for instance, but can we find such a basis for the ball? The answer is no, if we demand orthogonality, but what if we just ask for an unconditional (Riesz) basis? This is still unknown.

It was clear from the beginning that this question has a lot to do with tiling by translation (i.e., with filling up space with no overlaps by *translating* around an object). Fuglede originally conjectured that an orthogonal exponential basis exists if and only if the domain can tile space by translation. This has been disproved in its full generality but when one adds side conditions, such as, for instance, a lattice set of frequencies, or the space being a group of a specific type, or the domain being convex, or many other natural conditions, the answer is often unknown, and sometimes known to be positive or known to be negative. In short, this is a wide open area of research, branching out by varying the side conditions on the domain or the group in which the domain lives, or by relaxing the orthogonality condition or even allowing time-frequency translates of a given function to serve as basis elements (Gabor, or Weyl-Heisenberg, bases).

When working with both exponential bases and tiling problems the crucial object of study turns out to be the zero set of the Fourier Transform of the indicator function of the domain we care about. In particular we want to know how large structured sets this zero set contains, for instance how large difference sets it contains or what kind of tempered distributions it can support.

In this talk I will try to show how these objects are tied together, what has been done recently, and indicate specific open problems.

Guozhen Lu: *TBA.*

Misha Lyubich: *Quasisymmetries of Julia sets.*

We will describe groups of non-dynamical quasisymmetric homeos of Julia Sierpinski carpets, gaskets, and basilica-like. This is a joint project with Merenkov and Lodge.

Jack Milnor: *Two Moduli Spaces*

A discussion of two moduli spaces and their awkward topologies: first the space of divisors on the Riemann sphere modulo the action of Möbius automorphisms; and second the (compactified) space of curves in the complex projective plane modulo projective automorphisms. This is joint work with Araceli Bonifant.

Gerard Misiolek: *The L^2 exponential map in 2D and 3D hydrodynamics.*

In the 1960's V. Arnold showed how solutions of the incompressible Euler equations can be viewed as geodesics on the group of diffeomorphisms of the fluid domain equipped with a metric given by fluid's kinetic energy. The study of the exponential map of this metric is of particular interest and I will describe recent results concerning its properties as well as some necessary background.

Shahaf Nitzan: *TBA.*

Malabika Pramanik: *TBA.*

Ami Radunskaya: *Does noise help? Answers and more questions.*

Random fluctuations of an environment are common in ecological and economical settings. The processes describing the evolution of populations in these environments can often be described by a discrete, stochastic dynamical system, where a family of maps parametrized by a random variable forms the basis for a Markov Chain on a continuous state space. Random dynamical systems are a beautiful combination of deterministic and random processes, and they have received considerable interest since von Neumann and Ulam's seminal work in the 1940's. Key questions in the study of a stochastic dynamical system are: is there a unique, invariant measure? How does the long-term behavior compare to that of the state

variable in a constant environment with the averaged parameter?

In this talk we answer these questions for a family of maps on the unit interval that model self-limiting growth. The techniques used can be extended to study other families of concave maps, and so we state several generalizations of our results as conjectures. This is joint work with Peter Hinow, Mathematics Department, University of Wisconsin, Milwaukee.

Avraham Soffer: *Large Solutions to Supercritical Nonlinear Schrodinger equations.*

The construction of global solutions, for explicit classes of initial data, based on a new, microlocal and supercritical estimates will be presented. The cases considered are the H^1 - Supercritical , and the mass subcritical cases.

Parallel sessions

Ryan Alvarado: *A measure characterization of the Sobolev embedding theorem.*

Abstract: Historically, the Sobolev embedding theorem has played a key role in establishing many basic results in the area of analysis. Typically, sufficient conditions on the underlying measure have been imposed in order to guarantee the availability of the aforementioned theorem. In this talk, we will revisit this classical result and discuss some recent work which identifies a set of conditions on the measure that are both necessary and sufficient to ensure its veracity. This is joint work with Przemysław Górka (Warsaw University of Technology), Piotr Hajłasz (University of Pittsburgh).

Aleh Asipchuk, D.Dmitrishin, A.Stokolos: *Some applications of Fourier analysis in dynamical system theory.*

We will show some new applications of Fourier series multipliers to the problem of stability in discrete dynamical systems.

Alexander Barron, Jose Conde-Alonso, Yumeng Ou, and Guillermo Rey: *On Sparse Bounds for Certain Classical Bi-Parameter Operators.*

In recent years there has been a lot of activity related to “sparse bounds” for various operators in harmonic analysis. One demonstrates that certain operators can be controlled, in some sense, by a positive ‘sparse form’ involving averages of the input function over a sparse collection of cubes. Such an estimate yields easy proofs of quantitative weighted estimates which are often sharp.

Despite the recent activity, there has been virtually no progress in the context of bi-parameter operators such as the strong maximal function or bi-parameter Hilbert transform. We show that it is impossible to bound both of these operators by certain sparse forms involving L^1 and Orlicz-type averages over axis-parallel rectangles. The proof involves a construction of a sequence of point sets that are in some sense “extremal” for the bi-parameter structure. There are some connections between our arguments and discrepancy theory and probabilistic combinatorics.

Araceli Bonifant, Jack Milnor: *External rays for some families of cubic polynomial maps.*

We study the parameter space \mathcal{S}_p for cubic polynomials with a marked critical point of period p . We will show that for every escape region $\mathcal{E} \subset \mathcal{S}_p$, and every rational parameter angle ϕ , the parameter ray $\mathfrak{R}_{\mathcal{E}}(\phi)$ lands at some uniquely defined point in the boundary of the escape region $\partial\mathcal{E}$. This point is necessarily either critically finite or parabolic. Joint work with John Milnor.

Florin Catrina, Aurel Stan: *On the hypercontractivity of a convolution operator.*

We discuss a convolution operator which appears as an integral representation of the Wick product on $L^p(\mathbb{R}, \mu)$ spaces where the probability measure μ has a Gamma distribution. The hypercontractivity of this operator is tightly connected to inequalities of Brascamp-Lieb type.

Alan Chang: *The Kakeya needle problem for rectifiable sets.*

We show that the classical results about rotating a line segment in arbitrarily small area, and the existence of a Besicovitch and a Nikodym set hold if we replace the line segment by an arbitrary rectifiable set. This is joint work with Marianna Csörnyei.

Mark Comerford, Christopher Staniszewski: *The Amazing Universal Fatou Component.*

The possibilities for limit functions on a Fatou component for the iteration of a single polynomial or rational function are well understood and quite restricted. In non-autonomous iteration, where one considers compositions of arbitrary polynomials with suitably bounded degrees and coefficients, one should observe a far greater range of behaviour. We show this is indeed the case and we exhibit a bounded sequence of quadratic polynomials which has a bounded Fatou component on which one obtains as limit functions every member of the classical Schlicht family of normalized univalent functions on the unit disc. The main idea behind this is to make use of dynamics on Siegel discs where high iterates of a single polynomial with a Siegel disc U approximate the identity closely on compact subsets of U . Do almost nothing and you can do almost anything!

José Manuel Conde Alonso: *BMO from dyadic BMO for nonhomogeneous measures.*

The usual one third trick allows to reduce problems involving general cubes to a countable family. Moreover, this covering lemma uses only dyadic cubes, which allows to use nice martingale properties in harmonic analysis problems. In this talk, we consider alternatives to this technique in spaces equipped with nonhomogeneous measures. This entails additional difficulties which forces us to consider martingale filtrations that are not regular. The dyadic covering that we find can be used to clarify the relationship between martingale BMO spaces and the most natural BMO space in this setting.

Rachid El Harti: *The amenability to algebraic and analytical perspective.*

In this talk, we investigate the amenability to the point of algebraic and analytical view and its relationship with the semisimplicity in the case of operator algebras and crossed product Banach algebras associated with a class of \mathbb{C}^* -dynamical systems.

Behnam Esmayli, Piotr Hajlasz: *Holder Extension Of Maps Between Heisenberg Groups.*

Can we always extend a Lipschitz map initially defined from a subset of H^n to H^n to a globally defined Lipschitz map? (Here H^n denote the Heisenberg group.) Balogh, Lang, and Pansu answer this in the negative when n is odd. In this talk I will present a proof that the same holds for Holder maps of big enough Holder exponent. The named authors use Pansu's result on the differentiability of Lipschitz maps and the contact condition of such maps to show that globally defined Lipschitz maps from H^{2k+1} to itself must be orientation preserving. Contact condition guarantees that the horizontal plane maps to the horizontal under the derivative. Having this, it is then easy to construct counter-examples by alluding to degree theory. Unlike Lipschitz maps, Holder maps need not be differentiable, and there is no meaning to the contact condition. So, how does one begin? And that is what this talk will be!

Thomas Fallon: *The Fuglede Conjecture holds in (\mathbb{Z}_5^3) .*

The Fuglede Conjecture asserts that a bounded set $E \subset \mathbb{R}^d$ with positive Lebesgue measure tiles the space \mathbb{R}^d by translations if and only if $L^2(E)$ has an orthonormal basis of exponentials. The latter property means that E is a spectral set. Terry Tao disproved the conjecture in dimension $d = 5$. In dimensions

$d = 3$ and $d = 4$, the conjecture was disproved by disproved, in both direction, by Kolountzakis and Matolcsi. But, the one and two dimensional cases remained open.

The finite version of the Fuglede Conjecture on the finite vector spaces over the finite fields has been studied in special case. For example, Iosevich, Mayeli and Pakianathan proved that the conjecture holds for \mathbb{Z}_p^2 , p prime. In \mathbb{Z}_p^3 , the implication “tiling \Rightarrow spectral” was demonstrated by Kolountzakis and Matolcsi. It is not known whether spectral implies tiling in \mathbb{Z}_p^3 in general.

In this presentation, we discuss progress made relating to the Fuglede Conjecture in \mathbb{Z}_p^3 , and prove that the conjecture holds for \mathbb{Z}_5^3 . This is a joint work with Azita Mayeli and Dominick Villano.

Amineh Farzannia: *Biangular and two distance Harmonic Frames.*

Equiangular tight frames (ETFs) and biangular tight frames (BTFs) - sets of unit vectors with basis-like properties whose pairwise absolute inner products admit exactly one or two values, respectively - are useful for many applications. A well-understood class of ETFs are those which manifest as harmonic frames - vector sets defined in terms of the characters of finite abelian groups - because they are characterized by combinatorial objects called difference sets. This work is dedicated to the study of the underlying combinatorial structures of harmonic BTFs. We show that if a harmonic frame is generated by a divisible difference set or partial difference set - all two of which are generalizations of difference sets that fall under the umbrella term “bidifference set” - then it is either a BTF or an ETF.

Matthew Fleeman, Constanze Liaw: *Hyponormal Toeplitz Operators acting on the Bergman Space.*

We present some history on the study of hyponormal Toeplitz operators acting on the Bergman space, as well as give results for when the symbol is a non-harmonic polynomial. Particular attention is given to unusual hyponormality behavior that arises due to the extension of the class of allowed symbols.

Bryan Goldberg, Rongwei Yang: *Complex Dynamics on the Projective Spectrum of the Infinite Dihedral Group.*

Using the self-similarity of the infinite dihedral group (D_∞) in *Joint Spectrum and the Infinite Dihedral Group*, Grigorchuk and Yang defined a mapping $F : \mathbb{C}^3 \rightarrow \mathbb{C}^3$ where $F(z) = (z_0(z_0^2 - z_1^2 - z_2^2), z_1^2 z_2, z_2(z_0^2 - z_2^2))$. After establishing some background on $F(z)$ we'll use complex dynamics to establish some properties of this mapping. We'll use equivalent projective space and look at $F : \mathbb{P}^2 \rightarrow \mathbb{P}^2$ to discuss some results including the Fatou and Julia sets of $F(z)$ restricted to the projective spectrum. We'll conclude by examining connections between spectral theory and dynamics in this particular situation. This is joint work with Rongwei Yang.

Walton Green, Mishko Mitkovski, Shitao Liu: *Harmonic Analysis in Control Theory for PDEs.*

Harmonic analysis techniques have been applied to study control theory for PDEs from its inception in the late 60s by D.L. Russell. Estimates on exponential sums as well as interpolation theorems were originally applied to one-dimensional wave and heat equations. We have given the higher dimensional analogue for the observability of the wave equation and used this perspective to prove the same result for the visco-elastic wave equation (perturbation by a memory kernel). In this talk, I will present these results as well as relate them to equivalent statements concerning annihilating pairs for the Fourier transform.

Cristian Ivanescu: *The Cuntz semigroup and the classification of \mathbb{C}^* -algebras.*

An important class of \mathbb{C}^* -algebras (that announced by George Elliott in early 1990s) has been recently classified by means of K-theory. This class is referred to as the class of Z -stable \mathbb{C}^* -algebras. However examples of \mathbb{C}^* -algebras have been shown to exist outside of this class, requiring an enlargement of the

Elliott invariant. There is evidence that the Cuntz semigroup is useful in the classification theory. In this talk I will discuss the Cuntz semigroup as an invariant for \mathbb{C}^* -algebras and its applications to the classification theory.

Mohammad Jabbari, Donald Douglas, Xiang Tang, Guoliang Yu: *A New Index Theorem for Monomial Ideals by Resolutions*

I motivate and state an index theorem for the quotient module of a monomial ideal. This is obtained by resolving the monomial ideal by a sequence of essentially normal Hilbert modules, each of which is a direct sum of weighted Bergman spaces on unit balls.

Muzhi Jin: *On compactness of the $\bar{\partial}$ -Neumann operator on Hartogs domains*

We show that Property (P) of $\partial\Omega$, the $\bar{\partial}$ -Neumann operators N_1 , and compactness of Hankel operator on a smooth bounded pseudoconvex Hartogs domain $\Omega = \{(z, w_1, w_2, \dots, w_n) \in \mathbb{C}^{n+1} \mid \sum_{k=1}^n |w_k|^2 < e^{-2\varphi(z)}, z \in D\}$ are equivalent, where D is a smooth bounded connected open set in \mathbb{C} .

Spyridon Kakaroumpas, Sergei Treil: *Weighted estimates via “smooth” weights*

Hytönen’s celebrated A_2 theorem states that the norm of any Calderón-Zygmund operator over any weighted Lebesgue space $L^2(w)$ is linearly dominated by the A_2 characteristic of the weight w . This estimate is known to be sharp for “large” Calderón-Zygmund operators, such as the Hilbert transform. One can then ask whether this still holds if one replaces the usual A_2 characteristic by “fattened” A_2 characteristics which instead of averaging over intervals involve averaging against Poisson-like kernels. In this project we answer this and the analogous question for all $L^p(w)$ to the positive, by constructing examples with sufficiently “smooth” weights, meaning in this context weights having doubling constant sufficiently close to 2. We rely on ideas that F. Nazarov used to disprove Sarason’s conjecture in the two-weight setting for $p = 2$, addressing additional complications that arise due to having to construct one-weight examples rather than two-weight ones, and having to cover the entire range of p . This is joint work with Professor Sergei Treil (Brown University).

Ehssan Khanmohammadi: *A structured inverse spectrum problem for infinite graphs*

We present our extensions of some recent results on inverse eigenvalue problems of finite graphs to the infinite setting by means of functional analytic methods. Given an infinite graph G on countably many vertices and a closed, infinite set Λ of real numbers, we show, among other things, the existence of an unbounded operator whose graph is G and whose spectrum is Λ .

Damir Kinzebulatov: *$W^{1,p}$ regularity of solutions to Kolmogorov equation and associated Feller semigroup*

In \mathbb{R}^d , $d \geq 3$, consider the divergence and the non-divergence form operators

$$-\nabla \cdot a \cdot \nabla + b \cdot \nabla, \tag{i}$$

$$-a \cdot \nabla^2 + b \cdot \nabla, \tag{ii}$$

where $a = I + cf \otimes f$, the vector fields $\nabla_i f$ ($i = 1, 2, \dots, d$) and b are form-bounded (this includes the sub-critical class $[L^d + L^\infty]^d$ as well as vector fields having critical-order singularities). We characterize quantitative dependence on c and the values of the form-bounds of the $L^q \rightarrow W^{1,qd/(d-2)}$ regularity of the resolvents of the operator realizations of (i), (ii) in L^q , $q \geq 2 \vee (d-2)$ as (minus) generators of positivity preserving L^∞ contraction C_0 semigroups. The latter allows to run an iteration procedure $L^p \rightarrow L^\infty$ that yields associated with (i), (ii) L^q -strong Feller semigroups. This is joint work with Yu.A.Semenov (Toronto).

Chun-Kit Lai, Dorin Dutkay, Shahram Emami: *Exactness and Overcompleteness of Fourier frame for fractal measures.*

In this talk, we provide some recent progress on the construction of Fourier frames for fractal measures of convolutional type. In particular, the exactness and overcompleteness of Fourier frames will be discussed. Moreover, using a version of the recently proved Kadison-Singer theorem, we found that every self-similar measure admits exponential Riesz sequence/ Bessel sequence of maximal Beurling dimension. This is a joint work with D. Dutkay and S. Emami.

Jungang Li: *Trudinger-Moser Type Inequalities On Riemannian Manifolds.*

Abstract: In this talk three different Trudinger-Moser Type Inequalities on the Riemannian manifolds will be discussed. The first is the Chang-Marshall inequality on higher dimension, which could be viewed as a trace ainequality on a compact manifold. The second is a Trudinger-Moser type inequality on complete noncompact manifolds, where under certain curvature assumptions, a rearrangement-free argument will improve a local Trudinger-Moser inequality to a global result. The third is a Trudinger-Moser type inequality for maps between manifolds, which will give us useful information about harmonic maps. This is a joint work with Prof. Guozhen Lu.

Linhan Li, Jill Pipher: *Dirichlet problem for elliptic operators in divergence form with a BMO anti-symmetric part.*

We investigate the boundary behavior of solutions of divergence-form operators with an elliptic symmetric part and a BMO antisymmetric part. We establish the Holder continuity of the solutions at the boundary, existence of elliptic measure associated to such operators and the well-posedness of the L_p (w.r.t. elliptic measure) Dirichlet problem in non-tangentially accessible (NTA) domains. These general domains were introduced by Jerison and Kenig and include the class of Lipschitz domains. When specialized to Lipschitz domains, we are able to extend, to these operators, various criteria for determining mutual absolute continuity of elliptic measure with surface measure. For operators whose coefficients are independent of the vertical variable in the upper half space, we can show that the elliptic measure and surface measure are mutually continuous.

Tomas Merchan, Benjamin Jaye: *On the relation between L^2 boundedness and existence of principal value integral for a Calderón-Zygmund operator*

In 1998, Tolsa proved that any measure for which the Cauchy transform operator is bounded in $L^2(\mu)$ also exists in the sense of principal value. However, it turns out that this is not the case in general. Jaye and Nazarov created a measure in the complex numbers \mathbb{C} satisfying linear growth for which the singular integral operator with a simple kernel is bounded in $L^2(\mu)$ but fails to exist in the sense of principal value. In the talk, we will introduce sharp sufficient conditions on a measure which ensures that if a Calderón-Zygmund operator is bounded with respect to $L^2(\mu)$, then the operator exists in the sense of principal value. This is a joint work with Benjamin Jaye.

Soumyashant Nayak: *Jensen's inequality in finite subdiagonal algebras.*

Let M be a finite von Neumann algebra with a faithful normal tracial state τ and A be a finite subdiagonal subalgebra of M with respect to a τ -preserving faithful normal conditional expectation ϕ on M . Let Δ denote the Fuglede-Kadison determinant corresponding to τ . For $X \in M$, define $|X| := (X^*X)^{1/2}$. In 2005, Labuschagne proved the so-called Jensen's inequality for finite subdiagonal algebras i.e. $\Delta(\Phi(a)) \leq \Delta(a)$ for an operator $a \in A$, thus resolving a long-standing open problem posed by Arveson in 1967. In this talk, we will see that the following more general result holds: $\tau(f(|\Phi(a)|)) \leq \tau(f(|a|))$ for $a \in A$ and any increasing continuous function $f: [0, \infty) \rightarrow \mathbb{R}$ such that $f \circ \exp$ is convex on \mathbb{R} . Under the additional hypotheses that a is invertible in M and $f \circ \exp$ is strictly convex, we have $\tau(f(|\Phi(a)|)) = \tau(f(|a|)) \Leftrightarrow \Phi(a) = a$.

Josiah Park: *Lattices and Tight Frames.*

Lattice valued vector systems have taken an important role in packing, coding, cryptography, and signal processing problems. In compressed sensing, improvements in sparse recovery methods can be reached with an additional assumption that the signal of interest is lattice valued, as demonstrated by A. Flinth and G. Kutyniok. Equiangular tight frames are particular systems of unit vectors with minimal coherence, a measure of how well distributed the vectors are, and have provable guarantees for recovery of sparse vectors in standard methods. The determination whether real equiangular tight frames have integer span on a lattice has been given a characterization within two papers by A. Bottcher, L. Fukshansky, one with S. R. Garcia, H. Maharaj and D. Needell. Here the corresponding question is considered for the complex case and several families are demonstrated to have either integer span on lattice or not. In addition it is demonstrated that a real Parseval tight frame can have integer span on a lattice if and only if the inner products appearing in the system are rational (collaboration with L. Fukshansky, D. Needell, and X. Yuxin).

Arthur Parzygnat, Benjamin Russo: *Non-commutative disintegration.*

The notion of a disintegration of positive measures can be formulated diagrammatically in a category of transition kernels. Combining this with the functor relating transition kernels and positive operators, a notion of non-commutative disintegration can be made for certain C*-algebras and von Neumann algebras in terms of positive operators. While a certain degree of uniqueness holds as in the classical measure-theoretic case, existence of such disintegrations is not guaranteed even on finite-dimensional matrix algebras. Such disintegrations are closely related to reversible processes in quantum information theory and conditional probabilities in non-commutative probability. This is joint work with Benjamin P. Russo (Farmingdale State College SUNY).

José Pastrana: *Non-uniform Continuous Dependence of Data to Solution Map for Euler Equations in Besov Spaces.*

The Cauchy problem governing the motion of an incompressible fluid in a domain $\Omega \subseteq \mathbb{R}^d$ is given by

$$\begin{aligned}\partial_t u + (\nabla \cdot u)u + \nabla p &= 0; \\ \operatorname{div} u &= 0; \\ u(x, 0) &= u_0, \quad x \in \Omega, \quad t \in \mathbb{R}.\end{aligned}$$

Where $u : \mathbb{R} \times \Omega \rightarrow \mathbb{R}^d$ is the velocity field, $p : \mathbb{R} \times \Omega \rightarrow \mathbb{R}$ is the pressure function and $u_0 : \Omega \rightarrow \mathbb{R}^d$ is the divergence free initial data. We focus on $\Omega = \mathbb{R}^2/(2\pi\mathbb{Z}^2)$.

The local well-posedness theory has attracted a lot of attention over the years, see Majda and Bertozzi [11] and Bahouri, Chemin and Danchin [1]. In particular, there has been a lot of interest in the regularity properties of the data-to-solution map, $u_0 \rightarrow u$, of the Euler equations, ever since the papers of Kato and Ponce, see [8].

The use of approximate solutions technique traces back to Kenig, Ponce and Vega on KdV type equations, see [9]. Himonas and Misiolek [7] showed that the solution map is not uniformly continuous in H^s spaces, while Liu and Tang [10] generalized this result to the scales of periodic Besov spaces $B_{2,r}^s$. Cheskidov and Shvydkoy [5] showed that such map fails to be continuous at time $t = 0$ for the periodic Besov spaces $B_{r,\infty}^s$, and Bourgain and Li [2], [3], and [4] have proven strong ill-posedness in $W^{d/p+1,p}$, $B_{p,q}^{d/p+1}$, C^m , $C^{m-1,1}$.

Recently, Holmes, Keyfitz and Tiglay [6] used this method to show the non-uniform continuity of the solution map in H^s spaces for compressible gas dynamics. Motivated and following the ideas in [7] we exhibit two bounded sequences of solutions that converge at time $t = 0$ but remain apart later on. Thus, we show that continuity of such map is the best you can expect for the Besov space $B_{p,q}^s(\mathbb{T}^2)$ for

$s > 0$, $1 \leq p, q \leq \infty$ and the little Hölder class, $c^{1,\sigma}(\mathbb{T}^2) \subsetneq C^{1,\sigma}$, $\sigma \in (0, 1)$ where Misiolek and Yoneda [12], proved local well posedness in the sense of Hadamard.

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Nikolaos Pattakos, Leonid Chaichenets, Dirk Hundertmark, Peer Kunstmann: *Knocking out teeth in one-dimensional periodic NLS*.

We show the existence of weak solutions in the extended sense of the Cauchy problem for the cubic nonlinear Schrodinger equation in one dimension with initial data $u_0 \in H^s(\mathbb{R}) + H^r(\mathbb{T})$, $0 \leq s \leq r$. In addition, we show that if $u_0 \in H^s(\mathbb{R}) + H^{\frac{1}{2}+\epsilon}(\mathbb{T})$ where $\epsilon > 0$ and $\frac{1}{6} \leq s \leq \frac{1}{2}$ the solution is unique in $H^s(\mathbb{R}) + H^{\frac{1}{2}+\epsilon}(\mathbb{T})$.

Gabriel Prajitura, Ruhan Zhao, Jasbir Singh Manhas: *Frames and Operators*.

We will discuss certain relations between frames and operators. In particular we will look at frame preserving operators.

Timothy Rainone: *Finiteness and Dichotomy in \mathbb{C}^* -algebras.*

Notions of paradoxical decompositions appear in the work of Hausdorff, Banach, and Tarski who showed that a group satisfies the amenable/paradoxical divide. In this talk we study paradoxical phenomena in the field of operator algebras; restricting our attention to \mathbb{C}^* -algebras that arise from dynamical systems. Like Tarski, we use a type semigroup construction to pass from non-paradoxicality to the existence of traces. When the underlying algebra has a well-behaved K-theory, this semigroup witnesses the stably finite/purely infinite nature of the corresponding reduced crossed product \mathbb{C}^* -algebra. Moreover, we show that for a large class of these crossed product algebras stable finiteness is equivalent to being embeddable into a corona of matrix algebras.

David Renfrew: *Eigenvalues of random non-Hermitian matrices and randomly coupled differential equations*

We consider large random matrices with centered, independent entries but possibly different variances and compute the limiting distribution of eigenvalues. We then consider applications to long time asymptotics for systems of critically coupled differential equations with random coefficients.

Lev Sakhnovich: *Relativistic Lippmann -Schwinger equation (RLS equation)*

We constructed the RLS equation in the integral explicit form. With the help of the RLS equation we investigate spectral, stationary scattering and dynamical scattering problems for Dirac equation in the 3-dimensional space. We investigate the connections between the named problems.

Anamaria Savu: *Discrete solid-on-solid model.*

A crystal is a solid in which the atoms form a periodic arrangement. For many practical applications, understanding structural atomic arrangement and processes governing formation of crystals are essential to obtain useful properties. A special class of models so called Solid-on-Solid models are used to study the equilibrium statistical mechanics of surfaces. Several discrete Solid-on-Solid models and partial differential equations for surface diffusion are discussed.

Susanna Spektor: *The lower bound of the eigenvalues of Prolate Spheroidal Wave Function.*

I will show the best known lower bound for the eigenvalues of Prolate Spheroidal Wave Function.

Chris Staniszewski: *The Amazing Universal Fatou Component*

The possibilities for limit functions on a Fatou component for the iteration of a single polynomial or rational function are well understood and quite restricted. In non-autonomous iteration, where one considers compositions of arbitrary polynomials with suitably bounded degrees and coefficients, one should observe a far greater range of behaviour. We show this is indeed the case and we exhibit a bounded sequence of quadratic polynomials which has a bounded Fatou component on which one obtains as limit functions every member of the classical Schlicht family of normalized univalent functions on the unit disc. The main idea behind this is to make use of dynamics on Siegel discs where high iterates of a single polynomial with a Siegel disc U approximate the identity closely on compact subsets of U .

Robert Strichartz: *Two fun snapshots from harmonic anal.*

First: Suppose Fejer had been lazy, and instead of averaging all the partial sums of a Fourier series from 0 to N , he had averaged from a prescribed sparse collection of partial sums. Would uniform convergence still hold? In joint work with Ethan Goolish we found the answer to be sometimes yes, sometimes no, and in a lot of cases to be “most likely” with experimental evidence (nice pictures). Second: What do the eigenfunctions of the Laplacian on a regular polyhedron look like? In joint work with Evn Greif, Daniel Kaplan and Samuel Wiese, we found some beautiful pictures of them. It turns out that there are

nonsingular ones that are smooth at vertices, extend periodically to the plane, and are represented by trigonometric polynomials. There are also singular ones that are none of the above. The tetrahedron is boring because they are all nonsingular and lift to a double covering by a hexagonal torus. The octahedron is especially interesting because some nonsingular eigenfunctions can be rotated and dilated so that the eigenvalue is multiplied by $1/3$.

Scott Sutherland: *The measure of the Feigenbaum Julia Set.*

In joint work with Artem Dudko (IMPAN), we show that the Julia set of the quadratic Feigenbaum map has Hausdorff dimension less than two and consequently zero Lebesgue measure, answering a long-standing open question. This is established by a combination of new estimation techniques and a rigorous computer-assisted computation.

Feride Tiglay, John Holmes: *Data- to-solution map for the HunterSaxton equation in Besov spaces.*

The Cauchy problem for the HunterSaxton equation is known to be locally well posed in Besov spaces $B_{2,r}^s$ on the circle. We prove that the data-to-solution map is not uniformly continuous from any bounded subset of $B_{2,r}^s$ to $C([0, T]; B_{2,r}^s)$. We also show that the solution map is Hölder continuous with respect to a weaker topology.

Grennady Uraltsev: *Lebesgue points of the Fourier transform of a function and a maximal restriction estimate along variable directions.*

We discuss the result [1] of Müller, Ricci, Wright about the nature of the Lebesgue points of the Fourier transform of an L^p function and in particular if Fourier restriction to convex curves holds in the sense of Lebesgue differentiation. To do so we must study a maximal-type restriction operator.

We will show the boundedness of this operator in a more general setting than initially considered in [1]: we allow for a bi-parameter function in tangential and normal directions to the convex curve. This is based on a joint work with Marco Fraccaroli.

References

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Mai Tran, Rongwei Yang: *Non-Euclidean Metrics on the Resolvent Set.*

For a bounded linear operator A on a complex Hilbert space \mathcal{H} , the functions $g_x(z) = \|(A - z)^{-1}x\|^2$, where $x \in \mathcal{H}$ with $\|x\| = 1$, defines a family of non-Euclidean metrics on the resolvent set $\rho(A)$. Thus the arc length of a fixed circle $C \subset \rho(A)$ with respect to the metric g_x is dependent on the choice of x . This paper derives an integral equation for the extremal values of the arc length. If there exists a solution to the extremal equation, x , then it can be shown to have particular properties relating to A . In the case A is the unilateral shift operator on the Hardy space $\mathbf{H}^2(\mathbb{D})$, the paper proves that the arc length of C is maximal if and only if x is an inner function.

Ulises Velasco-García: *New methods for solving Sturm-Liouville equations applied to the Non-Linear Schrödinger Equation.*

In this talk we focus on the direct non-linear Fourier transform for the Non Linear Schrödinger Equation,

which reduces to the study of the Zakharov-Shabat (Z-S) system [1, 2, 3] of the form

$$\begin{pmatrix} v_1' \\ v_2' \end{pmatrix} = \begin{pmatrix} -i\lambda & q(x) \\ -q^*(x) & i\lambda \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix},$$

where $v_{1,2}$ are unknown complex functions, λ is the spectral parameter, the complex function $q(x)$ is the potential, $*$ is the complex conjugation and i is the imaginary unit. Since the Z-S system reduces to Sturm-Liouville equations we show, under a few restrictions for the potential, the spectral parameter power series [4] and the analytic approximation of transmutations operators [5] representations for the solutions of the Z-S system and the corresponding nonlinear Fourier coefficients. Finally we show numerical experiments, properties and numerical advantages of each method.

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Vyron Vellis: *Fractional rectifiability.*

Given a bounded set $E \subset \mathbb{R}^n$, when is it possible to construct a nice map (Holder, Lipschitz) from the unit interval into \mathbb{R}^n so that E is contained in its image? In this talk we discuss an extension of Peter Jones' traveling salesman construction, which provides a sufficient condition for E to be contained in a $(1/s)$ -Hölder curve, $s \geq 1$. The original result, corresponding to the case $s = 1$, identified subsets of rectifiable curves. When $s > 1$, $(1/s)$ -Hölder curves are more exotic objects than rectifiable curves that include snowflake curves and space-filling curves as basic examples. This talk is based on a joint work with Matthew Badger and a joint work with Matthew Badger and Lisa Naples.

Dominick Villano: *Radon-like transforms in intermediate dimension*

The mapping properties of Radon-like transforms are governed by the geometry of sub-manifolds in Euclidean space. In general, much more is known when these sub-manifolds are curves or hypersurfaces. In this talk, I will describe a technique that leverages the one dimensional theory to produce bounds for all dimensions.

Sven-Ake Wegner: *The heart of the Banach spaces.*

Let X be a Banach space and let Y be a linear subspace. If Y is closed in X then X/Y is a Banach space in the quotient norm. If Y is not closed then this is wrong—even if Y is a Banach space in a norm stronger than those induced by X . A prominent example is $Y = l^1$ and $X = c_0$. The unpleasant fact, that there

is no reasonable Banach space X/Y in the setting above, motivated Waelbroeck in the 1960s to consider formal quotients of Banach spaces. Amazingly, in 1982, the same year in which he published his paper on the category of quotient Banach spaces, Beilinson, Bernstein and Deligne published in a geometric context a very abstract, and by now very famous, theory about hearts of t-structures on triangulated categories. It turns out, that in their terminology, and for the special case of Banach spaces, the heart is precisely the category of formal quotients considered by Waelbroeck. In the talk we sketch the definition of the heart and discuss possible generalizations beyond the case of Banach spaces.

Yakun Xi: *Generalized periods estimates over curves on Riemannian surfaces.*

The generalized periods are the Fourier coefficients of eigenfunctions e_λ restricted to a closed smooth curve γ . In this talk, I will survey a few recent developments in the study of generalized periods. In particular, we show that the generalized periods over certain curves on non-positively curved surfaces would converge to zero at the rate of $O((\log \lambda)^{-1/2})$.

Ruhan Zhao: *Korenblum's Maximum Principle for the Bloch space.*

In this study we investigate the following problem: Given two analytic functions f, g in the Bloch space B on the unit disk D in the complex plane that satisfies $|f(z)| \leq |g(z)|$ for all $z \in D$. Is it true that $\|f\|_B \leq \|g\|_B$? We study this problem for polynomials, and show that, while the answer to this question is negative for certain pairs of polynomials, we do have certain cases that the answers are positive. Especially, we show that the above question has an affirmative answer if f and g are complex quadratics with $f(0) = g(0) = 0$.

Yujia Zhai, Yujia Zhai: *On the tensor product between a classical and a flag paraproduct.*

Classical and flag paraproducts arise naturally in the study of nonlinear PDEs. While multi-parameter paraproduct has been studied thoroughly, known estimates for flag paraproduct only involve single parameter. We prove L^p estimates for the tensor product between a classical and a flag paraproduct. We focus on the case when the bi-linear operator is restricted to tensor product of single-parameter functions and general bi-parameter functions. Such estimate implies a Leibniz rule with common presence in PDEs. A Loomis-Whitney type argument has been developed to derive the result.

Zhen Zeng: *Decay property of multilinear oscillatory integrals.*

In the groundbreaking work of Christ, Li, Tao and Thiele, they initial the study of conditions of the polynomial phase P and the linear mappings $\{\pi_i\}$ to ensure the power decay property of the corresponding multilinear oscillatory integrals. In this talk, I will extend their result to the trilinear case.

Poster abstracts

Anna Aboud, Emelie Curl, Nathan Harding, Mary Vaughan, Eric Weber: *The Dual Kaczmarz Algorithm.*

The Kaczmarz Algorithm is an iterative method for solving a system of linear equations. It can be extended so as to reconstruct a vector x in a (separable) Hilbert space from the inner-products $\langle x, \phi_n \rangle$. This extension uses the sequence $\{\phi_n\}$ in the reconstruction from the sequence $\langle x, \phi_n \rangle$, but only succeeds when the sequence is *effective*. We dualize the Kaczmarz Algorithm so that the reconstruction of x can be obtained from $\langle x, \phi_n \rangle$ by using a second sequence $\{\psi_n\}$ in the reconstruction. This allows for the reconstruction of x even when the sequence $\{\phi_n\}$ is not effective; in particular, our dualization yields a reconstruction when the sequence $\{\phi_n\}$ is *almost effective*. We also obtain some partial results characterizing when the reconstruction of x from $\langle x, \phi_n \rangle$ using $\{\psi_n\}$ succeeds, which we call an *effective*

pair.

Shiping Cao, Anthony Coniglio, Xueyan Niu, Richard Rand, Robert Strichartz: *The Mathieu Equation and Generalizations to Infinite Fractafolds.*

We study the Mathieu equation and its analog on the infinite Sierpinski Gasket. The Mathieu equation is a second order ordinary differential equation with two parameters. We study the stable and unstable areas in the parameter space, and experimentally see a “wedge” shape of the stable area. We also study periodic solutions on transition curves. We give an estimate on the convergence rate of Fourier coefficients of periodic solutions, and draw the periodic solutions experimentally. In addition, we define an analog of the Mathieu equation on the infinite Sierpinski Gasket with eigenvalue expansion. The periodic solutions and corresponding curves in the parameter spaces are studied. A similar convergence rate of Fourier coefficients of periodic solution applies, and solutions are drawn experimentally.

Simone Evans: *Asymptotic sets for networks with quadratic dynamics.*

While single-node quadratic networks have been studied over the past century, considering ensembles of such functions, organized as coupled nodes in an oriented network, generates new questions with potentially interesting applications to the life sciences. Many natural systems are organized as self-interacting networks composed of coupled quadratic nodes. Because these nodes receive functional input from not only themselves but also the other nodes in the network, they have ensemble behavior different from that of isolated functional nodes. Our objective is to study how the architecture of a network affects asymptotic dynamics. We extend accepted theorems and results from systems with isolated quadratic nodes to networks of quadratic nodes.

Tucker Lundgren: *TBA.*

Cassandra Williams: *Revising estimates of glutamate transporter density in astrocytes: a geometric computation.*

Glutamate is the main excitatory neurotransmitter released at chemical synapses in the brain. Its removal from the extracellular space is important to terminate synaptic transmission between neurons, and prevent neurotoxic build-up. The removal process is intermediated by astrocytes, the main class of non-neuronal cells in the brain. Astrocytes take in the excess extracellular glutamate via small, triangular-shaped glutamate transporters, densely expressed in the wall of their membrane. To understand their impact on neurotransmission efficiency, one needs to estimate the density of transporters for an average astrocyte. All existing computations are based on very simplifying assumptions of spherical shape for a typical cell. However, the actual, fractal-like 3-dimensional geometry of an astrocyte may drastically reduce this number, since transporters spanning through the cell boundary have to avoid colliding into each other. We are using a geometric argument, based on the known crystal structure of the glutamate transporter to determine, via a mathematical model, how the structural complexity of astrocytic processes influences the surface density of the transporters.